A Note on Ordering and Recovery Decision

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Abstract

Environment protection has attracted the attention all over the world nowadays. This study extends Koh's model to achieve an improved solution. A mathematical model is derived.

Keywords: Inventory, Recycling, EPQ

回收生產的改善策略

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摘 要

環保已是當今世界重要的議題，且引起許多學者注意。本研究主要在延伸 Koh 的回收生產模式。本文推導其數學模式以得改善的策略。

關鍵辭:存貨, 回收, 經濟生產量

1. Introduction

Environment protection has attracted the attention all over the world nowadays. One of the most important ways to protect our environment is recycling which can also reduce the production expense. According to Thierry et al. [1995] there are four reusable resources including direct reuse, repair, recycling, and remanufacturing. The main focus of this study is the handling of the reusable items with a simple recovery process. Schrady was the first one who established a model of the so-called reusable objects [1967]. Under the assumptions that demand and return rates are constant, external orders and recovery for production are fixed, and fixed costs (setup costs for orders and recovery process) together with linear holding costs for serviceable and recoverable inventory are the main concerns for total costs. Schrady developed a control strategy with fixed lot sizes serving demand from recovered products. Mabini et al. [1992] added two more factors, stockout service level constraints and multi-item system where items share the same repair facility. Richeter [1996] further clarified the difference between two models and described the relationship between the optimal control parameter values and return rate.

Koh et al. [2002] developed an ordering and recovery policy for reusable items based upon the EOQ and EPQ model. In their model they assumed to use recycled products with fixed return rate, through re-processing to provide the customers with a product as good a newly manufactured ones. Through the establishment of this process we are able to obtain an EOQ for newly procured products and the optimal inventory level of recoverable items to start the recovery process. But their assumption was “the model has one setup for recovery and many orders for new products” so that the solution will not necessarily be optimal. This study focuses on extending “the model has one setup for recovery and *n* orders for new products” to “the model has two setups for recovery and *n* orders for new products. Further expansion of our model into “many setups for recovery” for “optimal solution” in complicated problems requires more researches.

2. Mathematical model

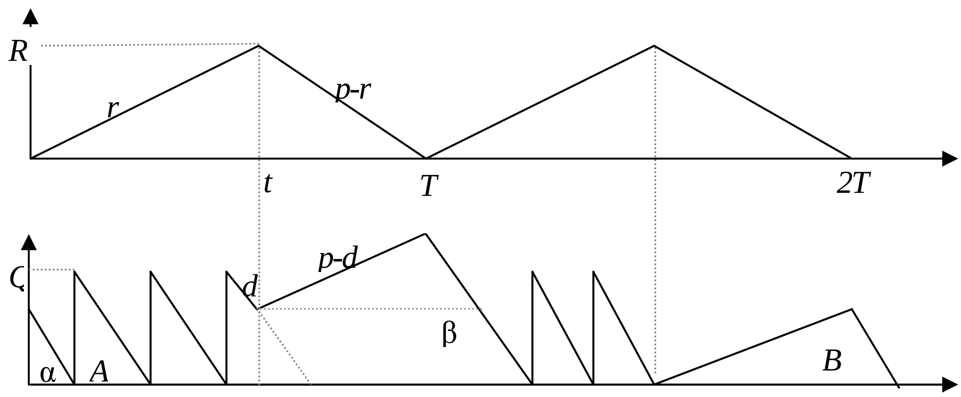

Fig.1 more orders for two recovery setups

Inventory model containing two types are depicted in Fig. 1:

(1) The recoverable process inventory model: During period “0~t” the product will be recycled once (being used twice) with the inventory level up to “R”.

(2) During “0~2T”, the new products will be ordered “2n+1” times (“2n” times is the same as Koh’s model) to satisfy the customers’ demand.

2.1. Assumptions and notations

Assumptions

1. The customers' demand is deterministic and known.
2. The recycle rate of used products from customers are known and fixed.
3. The quality of the recovered product is the same as the new product.
4. The repair capacity is constant and known.
5. The cost parameters are constant and known.
6. Shortages are not allowed.
7. The time required for external purchasing is fixed and known.
8. The time required for repair is fixed and known.
9. The economical efficacy of repairing items is higher than purchasing new product.
10. Recovery rate is greater than recycle rate ($p>r$).
11. Demand rate is greater than recycle rate ($d>r$).
12. Recovery rate is greater than demand rate ($p>d$).

Notations

Known parameters

r: recycle rate, (units/time)

p: recovery rate (units/time)

d: demand rate of the item, (units/time)

C_{h1}: inventory holding cost for the recoverable items,[$]/[unit]/[time]

C_{h2}: inventory holding cost for the serviceable items,[$]/[unit]/[time]

C_0: cost to order new product, [$]/[order]

C_S: setup cost for recovery process, [$]/[setup]

Unknown parameters

R: inventory level of recoverable items to start the recovery process

Q: order quantity for the newly product

2*n*+1: number of orders for two setups in the recovery shop where n is integer

2*m*-1: number of setups for two orders for new items where m is integer

T: cycle time

t: idle time length of the recovery process

2.2 Modeling analysis

In this section we addressed a situation of "2n+1" orders of the new product during the two recovery processes. The inventory process is demonstrated in Fig. 1.

In this model there will be "$2n + 1$" ($n = 1, 2, 3, \ldots$.) orders during two recovery processes so there will be "$n + 1$" orders during the first recovery processes with n orders during the second recovery processes (refer to Theorem 1). The inventory for the last cycle is

$$(p-d)(T-t).$$

with

$$Q=\frac{1}{2n+1}\left[d(T+t)-(p-d)(T-t)-p(T-t)\right].$$

so the inventory at "t" is

$$\frac{n+1}{2n+1}\left[d(T+t)-(p-d)(T-t)-p(T-t)\right]-d\left[t-\frac{(p-d)(T-t)}{d}\right].$$

The cost related to the recoverable inventory for two cycles is

$$2\left(C_S+\frac{C_{h1}}{2}RT\right).$$

with the cost for the serviceable items in the same period including the following:

(i) Ordering cost for 2n+1 orders,(n=1,2,3…)

$$(2n+1)C_O. \tag{1}$$

(ii) Inventory holding cost for 2n+1 right triangles (triangle A in Fig.1(i))

$$\frac{1}{2(2n+1)d}\left[d(T+t)-(p-d)(T-t)-p(T-t)\right]^2\cdot C_{h2}. \tag{2}$$

(iii) Inventory holding cost for 2 triangles (triangle B in Fig.1(i))

$$\left[\frac{(p-d)(T-t)}{d}+(T-t)\right]\cdot(p-d)(T-t)\cdot C_{h2}. \tag{3}$$

(iv) Inventory holding cost for parallelogram (middle area in Fig.1(i))

$$\left[\frac{(p-d)(T-t)}{d}+(T-t)\right]\cdot\left\{\frac{n+1}{2n+1}\left[d(T+t)-(p-d)(T-t)-p(T-t)\right]-d\left[t-\frac{(p-d)(T-t)}{d}\right]\right\}C_{h2}. \tag{4}$$

From Fig. 1 we know

$$T=\frac{pR}{r(p-r)}\quad,\quad\text{and} \tag{5}$$

$$t=\frac{R}{r}. \tag{6}$$

Now we can calculate the total cost per unit time by dividing the total cost of two cycles by 2T as follows:

$$TC(n,R)=\frac{2C_Spr-2C_Sr^2+pR^2C_{h1}+(2n+1)r(p-r)C_O}{2pR}+\frac{2pd^2-3pdr+2pr^2+2r^2pn-2r^2dn-r^2d}{2(2n+1)dr(p-r)}RC_{h2}\quad. \tag{7}$$

When "n" is fixed, the function is convex so the optimal can be obtained by differentiation of the function as follows:

$$R^* = \frac{\sqrt{-p(C_{h1}+2A_1)r(2nrC_O - 2pC_S + 2rC_S - 2npC_O - pC_O + rC_O)}}{(pC_{h1}+2pA_1)}.$$

where $A_1 = \dfrac{2pd^2 - 3pdr + 2pr^2 + 2r^2pn - 2r^2dn - r^2d}{2(2n+1)dr(p-r)} C_{h2}$. (8)

Because the other decision variable "*n*" (integer) in Eg.(7) can not be obtained so we used a simple numerical search procedure to resolve the variables.

When "*n*" and "*R*" are founded then the order quantity of the new product can be calculated as

$$Q = \frac{2p(d-r)}{(2n+1)r(p-r)} R. \quad (9)$$

Theorem 1. When $p>d>r$ with $2n+1$ orders (n=1, 2, 3, ..) between two recovery processes , then there will be (n+1) orders for the first recovery process and "*n*" orders for the second recovery process.

Proof: In Fig 2 the area of "β"must be larger than "α" (otherwise it will become a model of many orders for one recovery process), so the times of order during the first recovery process is higher than the second recovery process.

Claiming there are (n+1) orders during the first recovery process. When it is higher than (n+1) orders then the differences of orders between both processes will be higher than 3. It is no loss of generality to assume (n+2) orders during the first recovery process. If the n+2nd order during the first recovery process happens before "*t*" then the ordering during [b, t] will be higher than "n+1" whereas during [b_0, $t+T$] will be less than "(n-1) +1". In this situation there will be a conflict because the length of [b, T] will be the same as [b_0, $t+T$] so we prove the theorem.

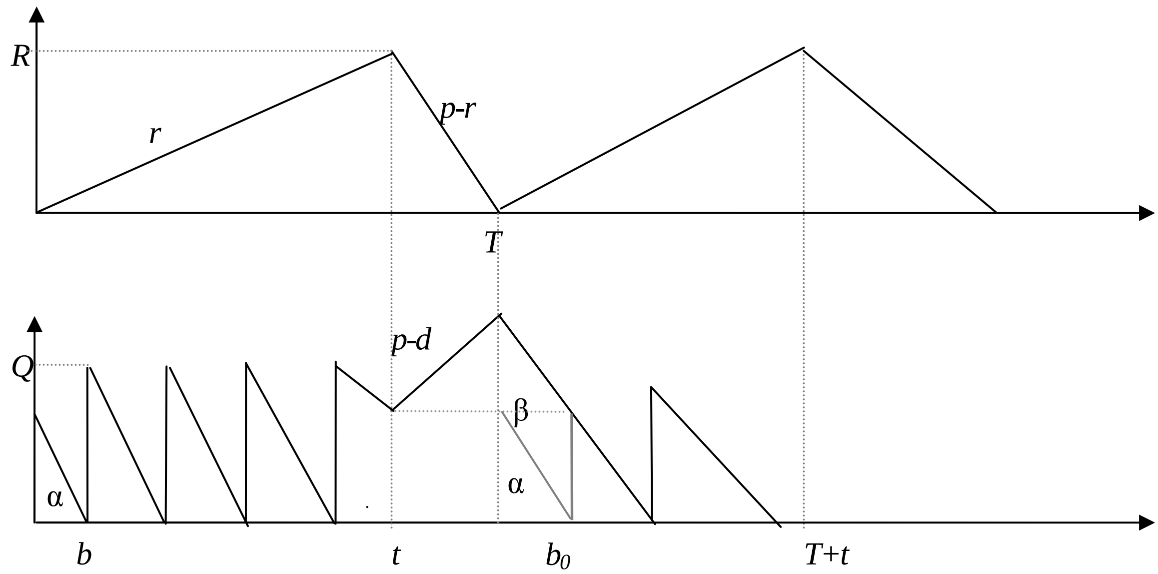

Fig.2 more orders for two recovery setups (detail).

3. Conclusion

Our study is trying to explore the condition of "many orders for two setups or many setups for two orders". The intention is to expand the foundation of the best solution. The critical steps during deduction of this model are expressed in theorem 1, 2, and 3. Also, it is very easy to apply our model into many orders for 3 or more setups. It is very complicate to expand our model into "k" setups and requires more researches. However, if we are able to confirm how many orders for each setup or how many setups for one order then it is not so difficult to figure out the answer.

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